Inverse Functions

Letting D and R indicate the domain and range of the respective function,

$$D_{f^{-1}} = R_f \text{ and } R_{f^{-1}} = D_f$$

Composite Functions

For a composite function gf(x) to exist, $R_f \subseteq D_g$

Domain of composite function gf(x): $D_{gf} = D_f$

Range of composite function gf(x): $R_{gf} \subseteq D_f$

if $R_f = D_g$, then $R_{gf} = R_g$

Sequence of Transformations

Transformations to a curve y=f(x) are to be carried out in the order a, b, c then d as below

d + cf(bx+a)

Conics

In general, the equation of the conic sections are represented by

$$Ax^2 + Cy^2 + Dx + Ey + F = 0, \text{ where } A\neq 0 \text{ and } C\neq 0$$

Circle	**Ellipse**	**Parabola**	**Hyperbola**
A = C	A ≠ C, AC > 0	A = 0 or C = 0	A≠C, AC < 0
$(x-h)^2+(y-k)^2=r^2$ $r\neq 0$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$y^2 = 4ax$ $x^2 = 4ay$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
centre at (h, k) radius is r	x-intercepts at (±a, 0) y-intercepts at (±b, 0)	foci at (a, 0) or (0, a)	Intercepts at (±a, 0) asymptotes at $y=\pm\frac{a}{b}$ or $y=\pm\frac{b}{a}$

Decomposition of Partial Fractions

$$\frac{px+q}{(ax+b)(cx+d)}=\frac{A}{(ax+b)}+\frac{B}{(cx+d)}$$

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2}=\frac{A}{(ax+b)}+\frac{B}{(cx+d)}+\frac{C}{(cx+d)^2}$$

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)}=\frac{A}{(ax+b)}+\frac{Bx+C}{(x^2+c^2)}$$

Arithmetic Progressions

$u_n=a+(n-1)d$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_n=\frac{n}{2}[a+u_n]$

Summation

$$\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3=\frac{1}{4}n^2(n+1)^2$$

Geometric Progressions

$u_n=ar^{n-1}$

If, $r\neq 1$ $S_n=\frac{a(r^n-1)}{r-1}=\frac{a(1-r^n)}{1-r}$

$S_\infty=\frac{a}{1-r}$, if $|r|<1$

DIFFERENTIATION

BASIC RULES

$$\frac{d}{dx}x^n=nx^{n-1}$$

$$\frac{d}{dx}[f(x)\pm g(x)]=f'(x)\pm g'(x)$$

$$\frac{d}{dx}n[f(x)]=n[f'(x)]$$

CHAIN RULE

$$\frac{d}{dx}f[g(x)]=f'[g(x)]g'(x)$$

e.g.

$f(x)=x^4$; $g(x)=x^3+6$

$$\frac{d}{dx}(x^3+6)^4=4(x^3+6)^3(3x^2)$$

QUOTIENT RULE

$$\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$$

"Low-d-high minus high-d-low over low squared"

PRODUCT RULE

$$\frac{d}{dx}[f(x)g(x)]=f'(x)g(x)+g'(x)f(x)$$

IMPLICIT DIFFERENTIATION

$$\frac{d}{dx}g(y)=\frac{dy}{dx}\cdot\frac{d}{dy}[g(y)] \qquad \text{e.g. } \frac{d}{dx}y^2=2y\frac{dy}{dx}$$

Standard rules of differentiation apply.

PARAMETRIC DIFFERENTIATION

For a parametric function x=f(t) and y=g(t)

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$$

INTEGRATION

BASIC RULES

$$\int(f\pm g)\,dx=\int f\,dx\pm\int g\,dx$$

$$\int n[f(x)]\,dx=n\int f(x)\,dx$$

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+c\ ,\ n\neq 1$$

$$\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+c\ ,\ n\neq 1$$

$$\int f'(x)\,[f(x)]^n dx=\frac{[f'(x)]^{n+1}}{n+1}\ ,\ n\neq 1$$

DEFINITE INTEGRALS

$$\int_a^b f(x)\,dx=\left[\int f(x)\,dx\right]_a^b$$

$$=F(b)-F(a)$$

Where F(x) is the antiderivative of f(x)

i.e., $\frac{d}{dx}F(x)=f(x)$

PARAMETRIC INTEGRATION

The integral of parametric function x=f(t) and y=g(t),

$$\int y\frac{dx}{dt}dt$$

AREA AND VOLUME

In general, the area R between two functions y=f(x) and y=g(x) with limits a to b is given by

$$\int_a^b y\,dx=\int_a^b f(x)-g(x)\,dx$$

And the volume found when R is revolved completely around the x-axis is given by

$$\pi\int_a^b y^2\,dx=\pi\int_a^b[f(x)]^2-[g(x)]^2\,dx$$

In general, the area Q between two functions x=f(y) and x=g(y) with limits c to d is given by

$$\int_c^d x\,dy=\int_c^d f(y)-g(y)\,dy$$

And the volume found when Q is revolved completely around the y-axis is given by

$$\pi\int_c^d x^2\,dy=\pi\int_c^d[f(y)]^2-[g(y)]^2\,dy$$

The area bound by the parametric function x=f(t) and y=g(t) and the x-axis, with limits e to f, is given by

$$\int_p^q y\frac{dx}{dt}dt$$

where f(p)=e and f(q)=f

Table of Derivatives

f(x)	f'(x)	f(x)	f'(x)	f(x)	f'(x)
sin x	cos x	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	e^x	e^x
cos x	-sin x			$e^{f(x)}$	$f'(x)[e^{f(x)}]$
tan x	$\sec^2 x$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$	ln x	$\frac{1}{x}$
csc x	-csc x cot x			ln [f(x)]	$\frac{f'(x)}{f(x)}$
sec x	sec x tan x	$\tan^{-1} x$	$\frac{1}{1+x^2}$	n^x	$n^x \ln n$
cot x	$-\csc^2 x$			$n^{f(x)}$	$n^{f(x)} f'(x)[\ln n]$

Table of Integrals

The arbitary constant c has been omitted

f(x)	$\int f(x)dx$	f(x)	$\int f(x)dx$	f(x)	$\int f(x)dx$
$\frac{f'(x)}{f(x)}$	ln |f(x)|	f'(x) cos[f(x)]	sin[f(x)]	$\frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}$	$\sin^{-1}\frac{f(x)}{a}+c$
		f'(x) sin[f(x)]	-cos[f(x)]		
$[f'(x)]e^{f(x)}$	$e^{f(x)}$	$f'(x)\sec^2[f(x)]$	tan[f(x)]	$\frac{f'(x)}{a^2+[f(x)]^2}$	$\frac{1}{a}\tan^{-1}\frac{f(x)}{a}+c$

Power Series

MacLaurin's Series

$$f(x) = f(0) + f'(0) + \frac{f''(0)x^2}{2!} + \frac{f^{(3)}(0)x^3}{3!} + \dots + \frac{f^{(n)}(0)x^n}{n!} + \dots$$

Small Angle Approximation

$$\sin\theta \approx \theta$$
$$\tan\theta \approx \theta$$
$$\cos\theta \approx 1-\frac{\theta^2}{2}$$

Complex Numbers

Forms of a Complex Number z

$$\begin{aligned} z &= x + iy \\ &= r(\cos\theta + i\sin\theta) \\ &= r e^{i\theta} \end{aligned}$$

Where $r=|z|$ and $\theta=\arg(z)$

Properties of Conjugate Pairs

$$\begin{aligned} (z^*)^* &= z \\ z + z^* &= 2\mathrm{Re}(z) \\ z - z^* &= 2i\,\mathrm{Im}(z) \\ zz^* &= x^2 + y^2 \end{aligned}$$

Multiplication of Complex Numbers

$$|z_1z_2| = |z_1||z_2| = r_1r_2$$
$$\arg(z_1z_2) = \arg(z_1) + \arg(z_2) = \theta_1+\theta_2$$

Division of Complex Numbers

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{r_1}{r_2}$$
$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) = \theta_1-\theta_2$$

de Moivre's Theorem

$$\begin{aligned} \text{if } z &= r(\cos\theta + i\sin\theta), \\ \text{then } z^n &= r^n(\cos\theta + i\sin\theta)^n \\ &= r^n(\cos(n\theta)+ i\sin(n\theta)) \end{aligned}$$

VECTORS

EQUATIONS OF A LINE

The vector equation of a line *l* which passes through pint A with position vector **a** and is parallel to a vector **b** is given by *l*: **r** = **a** + λ**b**

The line with Cartesian equation $\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}$ has a vector equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

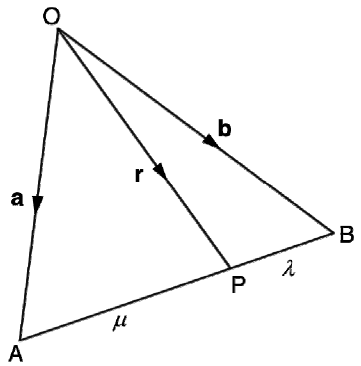
EQUATIONS OF A PLANE

The vector equation of a plane π which passes through pint A with position vector **a** and is parallel to two non parallel vectors **b** and **c** is given by π: **r** = **a** + λ**b** + μ**c**

The equation of the plan in scalar product form is given by π: **r.n** = p, where **r** is a position vector of any point on the plane π and **n** is the normal vector

The plane with Cartesian equation π: $n_1x + n_2y + n_3z = p$ has the equation $\mathbf{r}.\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = p$

RATIO THEOREM



For any point P that divides the line AB in the ratio λ:μ, the vector **r** is given by $\frac{\lambda a+\mu b}{\lambda+\mu}$

LENGTH OF PROJECTION

The length of projection of **a** onto **b** is given by

$$|a.\hat{b}|$$

SCALAR PRODUCT

$$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

where θ is the angle between the two vectors

Algebraically, $\begin{pmatrix} x_1 \\ y_2 \\ z_3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_2 \\ z_3 \end{pmatrix} = x_1x_2 + y_1y_2 + z_1z_2$

Important results to note are:

a.a = $|\mathbf{a}|^2$

a.a = 0 when the vectors are perpendicular

VECTOR PRODUCT

$$\mathbf{a} \times \mathbf{b} = (|a||b|\sin\theta)\,\hat{\mathbf{n}}$$

where θ is the angle between the two vectors

Algebraically, $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \times \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} y_1z_2 - z_1y_2 \\ -(x_1z_2 - z_1x_2) \\ x_1y_2 - y_1z_2 \end{pmatrix}$

FOOT OF PERPENDICULAR

From a point to a line

The foot, F, of the perpendicular of a point C with position vector **c** to a line with equation **r** = **a** + λ**b** is given by the equations relating

1) OF = **a** + λ**b**
2) OF.**b** = 0

From a point to a plane

The foot, Q, of the perpendicular of a point P with position vector **p** to a plane with normal **n** is given by the equations relating

1) OQ = **p** + λ**n**
2) OQ.**n** = p

PERPENDICULAR DISTANCE

In addition, the perpendicular distance from **a** to **b** is given by $|\mathbf{a} \times \hat{\mathbf{b}}|$