# PERMUTATIONS AND COMBINATIONS

The number of permutations of r objects, taken from a set of n distinct objects without replacement is given by

$$^{n}P_{r}=\frac{n!}{(n-r)!}$$

The number of permutation of r objects, taken from a set of n distinct objects with replacement is given by

$$n^{r}$$

The number of permutations of n distinct objects in a circle is given by

$$(n-1)!$$

The number of possible combinations of r objects, taken from a set of n distinct objects without replacement is given by

$$^{n}C_{r}=\frac{n!}{(n-r)!r!}$$

# PROBABILITY

## PROBABILITY

For two events A and B,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$P(A)=P(A\cap B)+P(A\cap B')$$

## MUTUAL EXCLUSIVITY

Mutually exclusive events cannot occur at the same time. For two mutually exclusive events, $E_1$ and $E_2$, $P(E_1\cup E_2)=P(E_1)+P(E_2)$

## CONDITIONAL PROBABILITY

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

## INDEPENDENCE

Independent events are events the occurrences of which do not influence the probability of the occurrence of the other event.

For independent events,

$$P(A|B)=P(A) \text{ or } P(B|A)=P(B)$$
$$P(A\cap B)=P(A)P(B)$$

# Random Variables

For any random variable X,

The **expectation**, μ, is given by

$$E(X)=\sum xP(X=x)$$

$$\text{and } E(aX\pm bY) = aE(X) \pm bE(Y)$$

The **variance** is given by

$$Var(X)=E[(X-\mu)^2]=\sum (x-\mu)^2 P(X=x)$$

$$\text{and } Var(aX\pm bY)=a^2 Var(X) + b^2 Var(Y)$$

The **standard deviation**, σ, is given by

$$\sigma=\sqrt{Var(X)}$$

# Discrete Random Variables

## Binomial Distribution

For a random variable X modelled by a binomial distribution with <u>n trials</u> and <u>probability of success, p</u>

$$X\sim B(n,p)$$

Its probability distribution is given by

$$P(X=x)={}^{n}C_{x}p^{x}(1-p)^{n-x}$$

Its mean and variance are given by

$$E(X) = np$$

$$Var(X) = np(1-p)$$

## Poisson Distribution

For a random variable X modelled by a Poisson distribution with <u>parameter λ</u>

$$X\sim Po(\lambda)$$

Its probability distribution is given by

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$$

Its mean and variance are given by

$$E(X) = Var(X) = \lambda$$

Note also that for two Poisson random variables

$$X\sim Po(\lambda_1) \text{ and } Y\sim Po(\lambda_2),$$

$X+Y\sim Po(\lambda_1+ \lambda_2)$

# Continuous Random Variables

## Normal Distribution

For a random variable X modelled by a normal distribution with <u>mean μ</u> and <u>standard deviation σ</u>

$$X\sim N(\mu,\sigma^2)$$

## Standard Normal Variable

Letting $X\sim N(\mu,\sigma^2)$, the standard normal variable Z is defined as

$$Z=\frac{x-\mu}{\sigma}\sim N(0,1) \text{ and}$$

$$P(X\leq x) = P\left(Z \leq \frac{x-\mu}{\sigma}\right)$$

# Approximations

Approximations marked † are to be continuity corrected

## Binomial to Poisson

For $X\sim B(n,p)$

If n is large ($n > 50$) and p is small ($p < 0.1$) such that **np < 5**, then $X\sim Po(np)$

## Binomial to Normal†

For $X\sim B(n,p)$

In is large such that **np > 5** and **n(1-p) > 5**, then $X\sim N(np,np(1-p))$

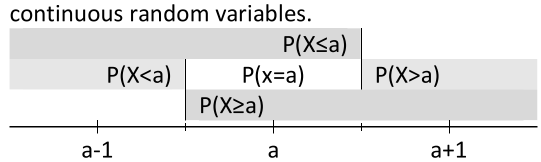
## Poisson to Normal†

For $X\sim Po(\lambda)$

If **λ > 10**, then $X\sim N(\lambda, \lambda)$

## Continuity Correction

These are the ranges, for given probability distribution functions, to consider when approximating discrete random variables to continuous random variables.



# SAMPLING

### SAMPLE MEAN

The sample mean from a normal population of sample size n with mean μ and variance $\sigma^2$ is given by

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

### UNBIASED ESTIMATOR OF SAMPLE MEAN

For any sample size n taken from a population with an unknown mean μ, the unbiased estimator of μ is given by

$$\bar{x} = \frac{\sum x}{n} = \frac{\sum (x-a)}{n} + a$$

where a is a constant

### CENTRAL LIMIT THEOREM

The central limit theorem states that, for a non-normal population with sample size n, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ approximately, if n is large (≥50).

### UNBIASED ESTIMATOR OF SAMPLE VARIANCE

For any sample size n taken from a population with an unknown mean $\sigma^2$, the unbiased estimator of $\sigma^2$ is given by

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right) = \frac{1}{n-1}\sum (x-\bar{x})^2$$

$$= \frac{1}{n-1}\left(\sum (x-a)^2 - \frac{\left(\sum (x-a)\right)^2}{n}\right)$$

where a is a constant

# HYPOTHESIS TESTING

### CONDUCTING A HYPOTHESIS TEST

Step 1: State the null and alternative hypotheses $H_0$ and $H_1$
Step 2: State the significance level, α
Step 3: Determine the test statistic to use and its distribution
Step 4: Calculate the p-value for the test statistic
Step 5: Indicate whether or not to reject $H_0$ based on the evidence from the sample
$H_0$ is rejected if p-value < α
$H_0$ is not rejected if p-value > α

### TEST STATISTICS

| | Normal Population | | Non-normal Population | |
|---|---|---|---|---|
| | $\sigma^2$ known | $\sigma^2$ unknown | $\sigma^2$ known | $\sigma^2$ unknown |
| Sample size is large n≥50 | $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ | $\bar{X} \sim N(\mu, \frac{s^2}{n})$ | by the CLT $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ | by the CLT $\bar{X} \sim N(\mu, \frac{s^2}{n})$ |
| Test Statistic | Z-test | Z-test | Z-test | Z-test |
| Sample size is small | $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ | $T \sim t(n-1)$ | | |
| Test statistic | Z-test | t-test | | |